

Exercise 58

Table 3 shows the input, w , and output, k , for a linear function k . a. Fill in the missing values of the table. b. Write the linear function k , round to 3 decimal places.

w	-10	5.5	67.5	b
k	30	-26	a	-44

Table 3

Solution

Begin by finding the equation of the line represented by this table. Use x for the input and y for the output so that the general equation of the line is

$$y = mx + b.$$

Two points on this line are $(-10, 30)$ and $(5.5, -26)$. The first point says that when $x = -10$, $y = 30$.

$$30 = m(-10) + b$$

The second point says that when $x = 5.5$, $y = -26$.

$$-26 = m(5.5) + b$$

This is a system of two equations with two unknowns that can be solved.

$$\begin{cases} -10m + b = 30 \\ 5.5m + b = -26 \end{cases}$$

Subtract the respective sides of these equations to eliminate b .

$$-10m - 5.5m = 30 - (-26) \quad \rightarrow \quad -15.5m = 56 \quad \rightarrow \quad m = -\frac{112}{31}$$

Multiply both sides of the second equation by $\frac{10}{5.5}$

$$\begin{cases} -10m + b = 30 \\ 10m + \frac{10}{5.5}b = \frac{-260}{5.5} \end{cases}$$

and then add the respective sides to eliminate m .

$$b + \frac{10}{5.5}b = 30 + \frac{-260}{5.5} \quad \rightarrow \quad \frac{15.5}{5.5}b = -\frac{190}{11} \quad \rightarrow \quad b = -\frac{190}{11} \times \frac{5.5}{15.5} = -\frac{190}{31}$$

Now that m and b have been solved for, the line is known.

$$y = -\frac{112}{31}x - \frac{190}{31}$$

When the input is 67.5, the output is

$$a = -\frac{112}{31}(67.5) - \frac{190}{31} = -250.$$

When the output is -44 , the input is

$$-44 = -\frac{112}{31}(b) - \frac{190}{31}$$

$$-44 + \frac{190}{31} = -\frac{112}{31}b$$

$$-\frac{1174}{31} = -\frac{112}{31}b$$

$$b = \frac{1174}{31} \times \frac{31}{112} = \frac{587}{56}.$$